Chapter 10

1

Rotation Of A Rigid Object About A Fixed Axis

1

Lecture 02

2

Rotational Kinematic Equations

2

Rotational Kinematics

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Under constant angular acceleration, we can describe the motion of the rigid object using a set of kinematic equations.

- These are similar to the kinematic equations for linear motion.
- The rotational equations have the same mathematical form as the linear equations.

The new model is a rigid object under constant angular acceleration.

- Analogous to the particle under constant acceleration model.

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Rotational Kinematic Equations

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Equations	Missing
$\omega_f = \omega_i + \alpha t$	$\Delta\theta$: displacement (m)
$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	t : time (s)
$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	ω_f : final velocity (m/s)
$\Delta\theta = \omega_f t - \frac{1}{2}\alpha t^2$	ω_i : initial velocity (m/s)
$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)t$	α : acceleration (m/s^2)

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Relationship Between Angular and Linear Quantities

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Every point on the rotating object has the same angular motion.

Every point on the rotating object does **not** have the same linear motion.

Displacements	• $s = r\theta$
Speeds	• $v = r\omega$
Accelerations	• $a = r\alpha$

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Speed Comparison – Details

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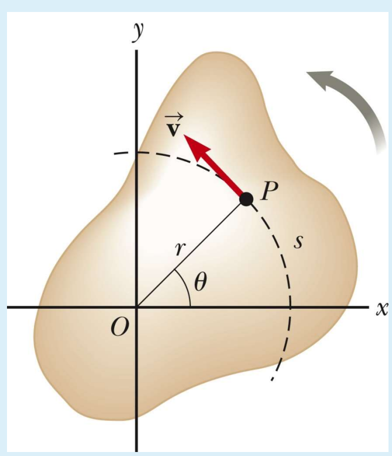
- The linear (tangential) velocity is always tangent to the circular path.
- The magnitude is defined by the tangential speed.

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$

- Since r is not the same for all points on the object, the tangential speed of every point is not the same.
- The tangential speed increases as one moves outward from the center of rotation.



6

Acceleration Comparison – Details

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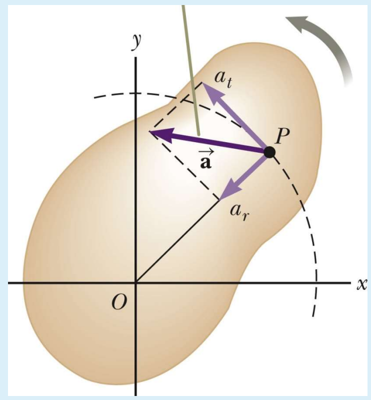
- The tangential acceleration is the derivative of the tangential velocity.

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$



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Speed and Acceleration Note

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- All points on the rigid object will have the same angular speed, but not the same tangential speed.
- All points on the rigid object will have the same angular acceleration, but not the same tangential acceleration.
- The tangential quantities depend on r, and r is not the same for all points on the object.

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Centripetal Acceleration

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An object traveling in a circle, even though it moves with a constant speed, will have an acceleration.

Therefore, each point on a rotating rigid object will experience a centripetal acceleration.

- $a_c = \frac{v^2}{r} = r\omega^2$

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Resultant Acceleration

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The tangential component of the acceleration is due to changing speed.

The centripetal component of the acceleration is due to changing direction.

Total acceleration can be found from these components:

- $a = \sqrt{a_t^2 + a_c^2} = r\sqrt{\alpha^2 + \omega^4}$

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Rotating Wheel 1

Saturday, 30 January, 2021 16:16

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A wheel rotates with a constant angular acceleration of $3.5\ rad/s^2$.

- If the angular speed of the wheel is $2\ rad/s$ at $t_i = 0$, through what angular displacement does the wheel rotate in $2\ s$?
- Through how many revolutions has the wheel turned during this time interval?
- What is the angular speed of the wheel at $t = 2\ s$?

$$\Delta\theta = \theta_f - \theta_i = \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\begin{aligned}\Delta\theta &= (2.00\text{ rad/s})(2.00\text{ s}) + \tfrac{1}{2}(3.50\text{ rad/s}^2)(2.00\text{ s})^2 \\ &= \boxed{11.0\text{ rad}} = (11.0\text{ rad})(180°/\pi\text{ rad}) = \boxed{630°}\end{aligned}$$

$$\Delta\theta = 630°\left(\frac{1\text{ rev}}{360°}\right) = \boxed{1.75\text{ rev}}$$

$$\begin{aligned}\omega_f &= \omega_i + \alpha t = 2.00\text{ rad/s} + (3.50\text{ rad/s}^2)(2.00\text{ s}) \\ &= \boxed{9.00\text{ rad/s}}\end{aligned}$$

Quadratic equation

Saturday, 30 January, 2021 16:17

A grindstone rotates at constant angular acceleration $0.35\ rad/s^2$. At time $t = 0$, it has an angular velocity of $-4.6\ rad/s$ and a reference line on it is horizontal, at the angular position $\theta_i = 0$. At what time after $t = 0$ is the reference line at the angular position $\theta_f = 5\ rev.$?

Calculations: Substituting known values and setting $\theta_0 = 0$ and $\theta = 5.0$ rev $= 10\pi$ rad give us

$$10\pi \text{ rad} = (-4.6 \text{ rad/s})t + \tfrac{1}{2}(0.35 \text{ rad/s}^2)t^2.$$

(We converted 5.0 rev to 10π rad to keep the units consistent.) Solving this quadratic equation for t, we find

$$t = 32 \text{ s.} \qquad \text{(Answer)}$$

Rotating Wheel 2

Saturday, 30 January, 2021 16:17

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

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H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A wheel accelerates uniformly from rest to an angular speed of $25\ rad/s$ in $10\ s$.

- Find the angular acceleration of the wheel.
- Find the tangential and radial acceleration of a point $10\ cm$ from the wheel's center.
- How many revolutions has the wheel turned during this time interval?
- Find the wheel's angular deceleration if it comes to a full stop after $5\ rev.$.

Solution: (a) We are given $\omega_\circ = 0$, $\omega = 25$ rad/s, and $t = 10$ s. To find the angular acceleration a, we can use $\omega = \omega_\circ + \alpha\ t$ as follows:

$$\alpha = \frac{\omega - \omega_\circ}{t} = \frac{25\,\text{rad/s} - 0}{10\,\text{s}} = 2.5\,\text{rad/s}^2$$

(b) Using Eqs. 8.13 and 8.14 (See Sect. 8.5), we get:

$$a_\text{t} = r\,\alpha = (10 \times 10^{-2}\,\text{m})(2.5\,\text{rad/s}^2) = 0.25\,\text{m/s}^2$$

$$a_\text{r} = r\,\omega^2 = (10 \times 10^{-2}\,\text{m})(25\,\text{rad/s})^2 = 62.5\,\text{m/s}^2$$

(c) If we assume that the wheel starts from $\theta_\circ = 0$, then we are given $\omega_\circ = 0$, $\omega = 25$ rad/s, $\theta_\circ = 0$, and $t = 10$ s. To find θ, which in this case equals the angle traveled by a certain reference line in the wheel, we use $\theta - \theta_\circ = \frac{1}{2}(\omega_\circ + \omega)\ t$ as follows:

$$\theta = \theta_\circ + \tfrac{1}{2}(\omega_\circ + \omega)\ t = 0 + \tfrac{1}{2}(0 + 25\,\text{rad/s}) \times 10\,\text{s} = 125\,\text{rad}$$

Thus:
$$\theta = 125\,\text{rad} \times \left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right) \simeq 20\,\text{rev}$$

(d) We are given $\omega_\circ = 25$ rad/s, $\omega = 0$, and, $\theta - \theta_\circ = 5\,\text{rev} = 10\pi$ rad. To find the angular deceleration α, we use $\omega^2 = \omega_\circ^2 + 2\ \alpha\ (\theta - \theta_\circ)$ to get:

$$\alpha = \frac{\omega^2 - \omega_\circ^2}{2\ (\theta - \theta_\circ)} = \frac{0 - (25\,\text{rad/s})^2}{2\ \times 10\pi\ \text{rad}} = -9.95\,\text{rad/s}^2$$